NONLOCAL REGULARIZATION FOR NON-ABELIAN GAUGE THEORIES FOR ARBITRARY GAUGE PARAMETER

Anirban Basu

and

Satish D. Joglekar

Department of Physics

I.I.T. Kanpur

Kanpur 208016, INDIA

ABSTRACT

We study the nonlocal regularization for the non-abelian gauge theories for an arbitrary value of the gauge parameter ξ. We show that the procedure for the nonlocalization of field theories established earlier by the original authors, when applied in that form to the Faddeev-Popov effective action in a linear gauge cannot lead to a ξ-independent result for the observables. We then show that an alternate procedure which is simpler can be used and that it leads to the S-matrix elements (where they exist) independent of ξ.

1.INTRODUCTION

Local Quantum Field Theories are plagued with infinities and need regularization to make the process of renormalization mathematically well-defined. Many regularizations have been proposed over the last 50 years, dimensional regularization being one used widest due to its effectiveness[1]. While dimensional regularization is useful in a wide class of Quantum Field Theories, it cannot be used directly in Supersymmetric Field Theories. A number of regularizations have been proposed over the last decade that can be used in Supersymmetric Field Theories[2,3]. Nonlocal regularization is one of them[2, 4, 5].

Nonlocal regularization proposed by Evans et al[2] has been extensively studied[4, 5, 6]. Renormalization procedure has been established upto two

loop order[5] in scalar theories. The scheme has found an elegant and neat formulation in reference 4 which has shown how nonlocally regularized field theories can be constructed from a local QFT in a systematic fashion. More importantly, it has been established that local/global symmetries can be preserved in their nonlocal form and the WT identities of local QFT's derivable from local symmetries such as gauge invariance/BRS symmetry find their natural nonlocal extensions. This has been done for the abelian gauge theories to all orders[7] and for nonabelian gauge theories in Feynman gauge[4] upto one loop order [limited only by the existence of measure beyond one loop].

Nonlocally regularized theories have also found other equally useful interpretations[4, 8]. Nonlocally regularized theories contain in them a large mass parameter Λ. It has been shown (wherever the measure factor exists) that these theories are unitary even with a finite Λ. Discussions of causality and renormalization group have also been carried out[4, 5]. Thus it has been suggested[8,9,10] such nonlocally regularized theories with a finite Λ can themselves be looked upon as valid physical theories (rather than a regularization for which $\Lambda \to \infty$ must be taken). The parameter Λ has been interpreted in two ways: (a) as a signal of an underlying space-time granularity; (b) as the mass scale beyond which the physical theory must be replaced by another, more fundamental theory. We may regard view (a) as a mathematically convenient way of embodying space-time granularity in QFT's in a way that is physically consistent. In view (b), we may regard the nonlocal QFT as an effective field theory that may have been derived from a more fundamental theory beyond the scale Λ. Thus, for example, we regard **nonlocal** standard model as the effective theory of fundamental processes at present energies, in which a signature of physics beyond standard model and the scale at which the SM should break down are both implicit in the scale Λ. An attempt to put lower bound using (g-2) of the muon has been made in Ref. 8.

The setting of such nonlocal QFT's has also been used to understand renormalization program in a mathematically rigorous way[10]. A way to put an upper bound on Λ has also been suggested[9, 10].

Nonlocal regularization has also found use in the discussion of higher loop anomalies in BV formulation[11].

In view of the above, it seems valuable to study these formulations further. One of the features of linear gauges in local gauge theories is the

availability of a free parameter ξ (gauge parameter) which helps in verifying the gauge independence of physical results. ξ-independence of physical results in spontaneously broken gauge theories has also been used to establish the cancellation of contributions from the unphysical poles to the cutting equations in SBGT[12].

It is therefore desirable that we have a formulation of non-local nonabelian gauge theories valid for an arbitrary ξ. Now, a well laid-out procedure for the nonlocalization of field theories has been presented in references 4 and 5. We found however that when we applied this procedure to the spontaneously broken theory (SM) in R_ξ gauges and calculated the (g-2) for the muon[8] we found a ξ-dependent result [13]. This motivated us to look into the question of nonlocal formulation of unbroken nonabelian gauge theories and of spontaneously broken chiral abelian gauge theories[14]. In the present work, we concern ourselves with the former.

We now discuss the plan of our work. In Section II, we summarize the results on the nonlocal quantum field theories of 2, 4, 5. In Section III, we adopt the procedure outlined in 4 for nonlocalization for arbitrary ξ and evaluate $\xi\frac{dW}{d\xi}|_{\xi=1}$for this case. We obtain a term in $\xi\frac{dW}{d\xi}|_{\xi=1}$ that **can** contribute to on-shell physical processes and which cannot be cancelled by a ξ dependent measure. In Section IV, we suggest an alternate way of constructing nonlocal unbroken gauge theories for an arbitrary ξ and establish the WT identity satisfied by the physical Green's function $\xi\frac{dW}{d\xi}|_{J=J_{phy}}$ of eqn. (4.15). This equation is analogous to that in that in the local case; and should lead to the ξ-independence of physical quantities that are free of infra-red divergences.

II REVIEW OF KNOWN RESULTS

A Nonlocal Regularization

Let us briefly review the method of non-local regularization as proposed in [4, 5]. Let ϕ_i stand for a generic, not necessarily scalar, field and let us assume that the local action can be written as a standard free part plus an interaction

$$S[\phi] = F[\phi] + I[\phi] \ , \tag{2.1}$$

where

$$\mathrm{F}[\phi] = \frac{1}{2} \int d^D x \ \ \phi_i(x)\ \mathcal{F}_{ij} \ \ \phi_j(x) \tag{2.2}$$

Here, $\mathcal{F}$ is the kinetic energy operator for the field ϕ, and I$[\phi]$ is the interaction term. For unbroken gauge theories, S$[\phi]$ would be the BRS gauge fixed action and ϕ_i would include both the fields of the invariant action and the ghosts introduced in the process of fixing the gauge.

From the kinetic energy operator $\mathcal{F}$, let us define a non-local smearing operator $\mathcal{E}$ and a shadow kinetic operator $\mathcal{O}^{-1}$ as[1]

$$\mathcal{E} = \exp[\tfrac{\mathcal{F}}{2\Lambda^2}] \tag{2.3}$$

and

$$\mathcal{O} = \frac{\mathcal{E}^2 - 1}{\mathcal{F}} \tag{2.4}$$

Further, the smeared field is defined as

$$\hat{\phi} = \mathcal{E}^{-1}\ \phi \tag{2.5}$$

For every field ϕ, an auxiliary field ψ of the same type is introduced. Then, the auxiliary action is defined to be

$$\mathcal{S}[\phi,\psi] = \mathrm{F}[\hat{\phi}] - \mathrm{A}[\psi] + \mathrm{I}[\phi+\psi] \tag{2.6}$$

where

$$\mathrm{A}[\psi] = \frac{1}{2}\int d^D x \ \ \psi_i(x)\ \mathcal{O}^{-1}_{ij} \ \ \psi_j(x) \tag{2.7}$$

The action for the nonlocalized theory $\hat{S}[\phi]$ is defined to be

$$\hat{S}[\phi] = \mathcal{S}[\phi,\psi[\phi]] \tag{2.8}$$

where $\psi[\phi]$ is a solution of the classical shadow field equation

[1] In the case of the ghost Lagrangian, its overall sign (and therefore that of the quadratic form) is arbitrary. In such a case, we shall always take the sign of $\mathcal{F}$ such that the ghost propagator is damped for large $|k^2|$ in Euclidean space.

$\frac{\delta \mathcal{S}[\phi,\psi]}{\delta \psi_i} = 0 \qquad (2.9)$

Quantization is carried out in the path integral formulation. The quantization rule is

$< T^*(O[\phi]) >_{\mathcal{E}} = \int [D\phi]\mu[\phi]O[\hat{\phi}]e^{i\hat{S}[\phi]} \qquad (2.10)$

Here O is any operator taken as a functional of fields. $\mu[\phi]$ is the measure factor defined such that $[D\phi]\mu[\phi]$ is invariant under the nonlocal generalisation of the local symmetry. For nonlocalized non-Abelian gauge theories, this measure factor can be non-trivial and has been evaluated upto one loop [4]. For the abelian gauge theories, the measure factor is known to all orders[7].

The nonlocalized Feynman rules are simple extensions of the local ones. The vertices are unchanged but every leg can connect either to a smeared propagator

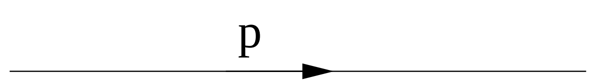
$\frac{i\mathcal{E}^2}{\mathcal{F}+i\epsilon} =$ -i$\int_1^\infty \frac{d\tau}{\Lambda^2} e^{\frac{\mathcal{F}\tau}{\Lambda^2}} \qquad (2.11)$

or to a shadow propagator

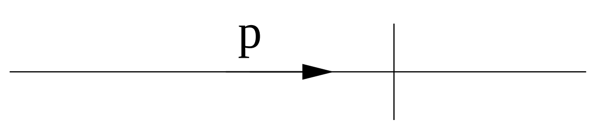
$\frac{i(1-\mathcal{E}^2)}{\mathcal{F}} =$ -i$\mathcal{O} =$ -i$\int_0^1 \frac{d\tau}{\Lambda^2} e^{\frac{\mathcal{F}\tau}{\Lambda^2}} \qquad (2.12)$

Diagramatically, they will be represented as

the smeared or "unbarred" propagator



the shadow or "barred" propagator



The shadow propagator lacks a pole and so carries no quanta. Thus, the following points are to be noted:

i) All external lines must be unbarred.
ii) The symmetry factor for any diagram is computed without distinguishing between barred and unbarred lines.
iii) The loop integrations are well defined in the Euclidean space because of the exponential damping factors coming from propagators within loops.
iv) Internal lines can be smeared or barred. However, loops containing only the shadow lines are forbidden.
v) Tree order Green's functions are unchanged except for external line factors which are unity on shell. This follows because every internal line of a tree graph can be either barred or unbarred. Hence, it is the sum of both these that which enters, which gives the local propagator.

B Theorems Regarding Nonlocal Regularized Actions

Before discussing the nonlocal BRS symmetries, let us consider a few theorems concerning classical solutions of the Euler-Lagrange equations associated with the local action S$[\phi]$, the auxiliary action $\mathcal{S}[\phi,\psi]$ and the nonlocalized $\hat{S}[\phi]$ action:

Theorem A.1: The shadow fields can be expressed as follows:

$$\psi_i[\phi] = -(\tfrac{\mathcal{E}^2-1}{\mathcal{E}^2})_{ij}\phi_j + O_{ij}\tfrac{\delta\hat{S}[\phi]}{\delta\phi_j} \qquad (2.13)$$

Theorem A.2: If ϕ_i and ψ_i obey the Euler-Lagrange equations of $\mathcal{S}[\phi,\psi]$ then $\chi_i = \phi_i + \psi_i$ obeys the Euler-Lagrange equations of S$[\chi]$.

Theorem A.3: If χ_i obeys the Euler-Lagrange equations of S$[\chi]$, then the following fields

$$\begin{aligned}\phi_i &= \mathcal{E}^2_{ij}\chi_j \qquad (2.14)\\ \psi_i &= (1-\mathcal{E}^2)_{ij}\chi_j\end{aligned}$$

obey the Euler-Lagrange equations of $\mathcal{S}[\phi,\psi]$.

Let us also consider another set of theorems concerning classical symmetries of S[χ], $\mathcal{S}[\phi, \psi]$ and $\hat{S}[\phi]$:

Theorem B.1: If S[ϕ] is invariant under the infinisitesimal transformation

$\delta\phi_i = T_i[\phi]$,

then the following transformation is a symmetry of $\mathcal{S}[\phi, \psi]$:

$$\Delta\phi_i = \mathcal{E}^2_{ij} T_j[\phi + \psi] \qquad (2.15)$$
$$\Delta\psi_i = (1 - \mathcal{E}^2)_{ij} T_j[\phi + \psi].$$

Theorem B.3[2]: If S[ϕ] is invariant under $\delta\phi_i = T_i[\phi]$, then $\hat{S}[\phi]$ is invariant under

$$\hat{\delta}\phi_i = \mathcal{E}^2_{ij} T_j[\phi + \psi[\phi]] \qquad (2.16)$$

TheoremB.4: The following transformation generates a symmetry of $\mathcal{S}[\phi, \psi]$:

$$\Delta\phi_i = A_{ij}[\phi, \psi]\{\tfrac{\delta\mathcal{S}[\phi,\psi]}{\delta\phi_j} - \tfrac{\delta\mathcal{S}[\phi,\psi]}{\delta\psi_j}\} \qquad (2.17)$$
$$\Delta\psi_i = \text{-}A_{ij}[\phi, \psi]\{\tfrac{\delta\mathcal{S}[\phi,\psi]}{\delta\phi_j} - \tfrac{\delta\mathcal{S}[\phi,\psi]}{\delta\psi_j}\}$$

provided $A_{ij}[\phi, \psi] = \text{-}A_{ji}[\phi, \psi]$. This symmetry is a "trivial symmetry" without a dynamical content.

$\Delta\phi_i$ can also be cast in the simple form

$$\Delta\phi_i = A_{ij}[\phi, \psi]\mathcal{E}^{-2}_{jk}\mathcal{O}^{-1}_{kl}[(1 - \mathcal{E}^2)_{lm}\phi_m - \mathcal{E}^2_{lm}\psi_m] \qquad (2.18)$$

An important special case is given by the choice

$A_{il}[\phi, \psi] = M_{ij}\mathcal{O}_{jk}\mathcal{E}^2_{kl}$

where

[2]We stick to the theorem numbering of [4].

$[M_{ij}, \mathcal{F}_{ij}] = 0$ and $M_{ij} = -M_{ji}$.

We next review the nonlocal BRS symmetries of the Yang-Mills theory using the above results.

C Nonlocal Regularization of Yang-Mills Theory in the Feynman Gauge

Finally, let us consider the nonlocal regularization of Yang-Mills theories. First, we will study the results obtained in the Feynman gauge.

The Feynman gauge local BRS Lagrangian is

$$\mathcal{L}_{BRS}=-\tfrac{1}{2}\partial_\mu A^a_\nu\,\partial^\mu A^{a\nu}-\partial_\mu\overline{\eta}^a\,\partial^\mu\eta^a+g\,f^{abc}\partial_\mu\overline{\eta}^a\,A^{b\mu}\eta^c+g\,f^{abc}\partial_\mu A^a_\nu\,A^{b\mu}A_{c\nu}$$
$$-\tfrac{g^2}{4}f^{abc}f^{cde}A^a_\mu A^b_\nu A^{d\mu}A^{e\nu} \qquad (2.19)$$

Thus, the gluon and the ghost kinetic energy operators are $\mathcal{F}^{\mu\nu}_{ab} = \delta_{ab}\eta^{\mu\nu}\partial^2$ and $\mathcal{F}_{ab} = -\delta_{ab}\partial^2$ respectively[3].

Let us denote the auxiliary fields of A^a_μ and η^a by B^a_μ and ψ^a respectively.

Thus the non-localized BRS action is

$$\hat{S}[\mathrm{A},\eta,\overline{\eta}]=\int d^4x\{-\tfrac{1}{2}\partial_\mu\hat{A}^a_\nu\,\partial^\mu\hat{A}^{a\nu}-\tfrac{1}{2}B^a_\mu O^{-1}B^{a\mu}-\partial_\mu\hat{\overline{\eta}}^a\,\partial^\mu\hat{\eta}^a+\psi^a O^{-1}\psi^a\}$$
$$+\mathrm{I}[\mathrm{A}+\mathrm{B},\eta+\psi,\overline{\eta}+\overline{\psi}] \qquad (2.20)$$

The local BRS Yang-Mills action in the Feynman gauge has the following BRS symmetry transformations:

$$\delta A^a_\mu = (\partial_\mu\eta^a - g f^{abc}A^b_\mu\eta^c)\delta\varsigma$$

$$\delta\eta^a = -\tfrac{g}{2}f^{abc}\eta^b\eta^c\delta\varsigma \qquad (2.21)$$

$$\delta\overline{\eta}^a = -\partial_\mu A^{a\mu}\delta\varsigma$$

where $\delta\varsigma$ is a constant anticommuting C number.

[3] See earlier footnote above eq.(2.3).

Given the local symmetry transformations for the fields, one can easily write down their non-local counterparts:

$$\hat{\delta}A^a_\mu = \mathcal{E}^2[\partial_\mu(\eta^a+\psi^a) - gf^{abc}(A^b_\mu + B^b_\mu)(\eta^c+\psi^c)]\delta\varsigma$$

$$\hat{\delta}\eta^a = \text{-}\tfrac{g}{2}f^{abc}\mathcal{E}^2(\eta^b+\psi^b)(\eta^c+\psi^c)\delta\varsigma \qquad (2.22)$$

$$\hat{\delta}\overline{\eta}^a = -\mathcal{E}^2\partial_\mu(A^{a\mu}+B^{a\mu})\delta\varsigma$$

where

$$\mathcal{E} = e^{\frac{\partial^2}{2\Lambda^2}}$$

In [4], it was found that it is convenient to construct a modified nonlocal BRS symmetry transformation by adding a "trivial" symmetry transformation to the kind (2.18). This was so as noted in [4] since it yielded a variation of $\overline{c}$ proportional to ∂.A. Put alternatively, we find that these new transformations of [4] have two useful properties (i) $\partial.\delta$A is directly reducible in terms of the ghost action, (ii) WT identities so formulated allow an easy evaluation of $\xi\frac{\partial W}{\partial\xi}$. The measure factor is defined with respect to the latter transformations. They are

$$\hat{\delta}A^a_\mu = [\partial_\mu\eta^a - gf^{abc}\mathcal{E}^2(A^b_\mu + B^b_\mu)(\eta^c+\psi^c)]\delta\varsigma$$

$$\hat{\delta}\eta^a = \text{-}\tfrac{g}{2}f^{abc}\mathcal{E}^2(\eta^b+\psi^b)(\eta^c+\psi^c)\delta\varsigma \qquad (2.23)$$

$$\hat{\delta}\overline{\eta}^a = -\partial_\mu A^{a\mu}\delta\varsigma$$

The measure factor[4] is

$$\ln(\mu[A,\eta,\overline{\eta}]) = \text{-}\frac{g^2}{2}f_{acd}f_{bcd}\int d^Dx A_{\mu a}\mathcal{M}A_b{}^\mu + \mathrm{O}(g^3) \qquad (2.24)$$

where

$$\mathcal{M} = \frac{1}{2^D\pi^{\frac{D}{2}}}\int_0^1 d\tau\frac{\Lambda^{D-2}}{(\tau+1)^{\frac{D}{2}}}\ \exp\left(\frac{\tau}{\tau+1}\frac{\partial^2}{\Lambda^2}\right)\left[\frac{2}{\tau+1} - (D-1) + 2(D-2)\frac{\tau}{\tau+1}\right]$$

III Difficulty with the method of non-local regularization for an arbitrary ξ:

The above method of regularization worrks correctly in the Feynman gauge ξ=1. The regulator operators are simple and calculations have been performed in this gauge with relative ease [4, 8]. When this procedure of taking the entire quadratic form $\mathcal{F}$ which enters the regulator operators is used, the regulators are ξ-dependent and complicated. This, of course, is not a serious objection to the use of this procedure in [4, 5], we find that the procedure in fact leads to WT identities which imply that the S-matrix elements, where they exist, are not ξ-independent even in one loop order. In this section, we wish to demonstrate it and then suggest, in the next section, an alternate way of regularization which is at once simpler and leads to a WT identity which formally implies the ξ-independence of the S-matrix.

An abelian special case of this has already been applied to QED[7]. Originally we derived the motivation for this work from the following observation in the context of the SM (where physical S-matrix elements generally exist). We had found ξ-dependence of the muon anomalous magnetic moment in the SM in [8] when we follow the procedure of references[4, 5] as applied to the spontaneously broken (local) theory for the R_ξ gauges[13]. The discussion given here in this work has also been extended [14] to the spontaneously broken U(1) chiral model where a similar ξ-dependence of a physical quantity has neen demonstrated. The procedure, formulated here in section IV has been applied there to this case; with formal ξ-independence established[14].

We now consider the non-local action for an arbitrary ξ. Here, we will generalize (following [4, 5]), for an arbitrary ξ, the appropriate non-local action. We express

$$\hat{S}_\xi=\hat{S}+\Delta\hat{S}, \qquad (3.1)$$

where

$\hat{S}$=$\int d^4x(-\frac{1}{2}\partial_\mu \hat{A}^a_\nu\, \partial^\mu \hat{A}^{a\nu} - \partial_\mu \hat{\overline{\eta}}^a\, \partial^\mu \hat{\eta}^a - \frac{1}{2} B^a_\mu \mathcal{O}_A^{-1ab\mu\nu} B^b_\nu + \overline{\psi}^a \mathcal{O}_\eta^{-1ab} \psi^b$
+Interaction terms) (3.2)

and

$\Delta\hat{S}$=$\frac{1}{2}(1-\frac{1}{\xi}) \int d^4x(\partial.\hat{A}^a)^2$ (3.3)

We note that as $\Lambda \to \infty$, $\hat{S}$ reduces to the local action of the Feynman gauge. Note, now, that the smeared gauge field $\hat{A}$ has been constructed using the full (ξ dependent) quadratic form $\mathcal{F}$

$$\hat{A}_\mu = \mathcal{E}^{-1}_{A\mu\nu} A^\nu = (e^{-\frac{\mathcal{F}}{2\Lambda^2}})_{\mu\nu} A^\nu \qquad (3.4)$$

and the ghost fields $\hat{\eta}$ and $\hat{\overline{\eta}}$ have been smeared using their respective quadratic forms

$$\hat{\eta} = \mathcal{E}_\eta^{-1}\eta \qquad \hat{\overline{\eta}} = \mathcal{E}_\eta^{-1}\overline{\eta} \qquad \mathcal{E}_\eta = e^{\frac{-\partial^2}{2\Lambda^2}} \qquad (3.5)$$

where it should be noted that $\mathcal{E}_\eta$ is independent of ξ.

We shall now proceed to evaluate $\xi\frac{\partial W}{\partial \xi}|_{\xi=1}$ for the above nonlocal theory. We note that the ξ dependence of W comes from (i) the explicit ξ dependence of $\Delta\hat{S}$ (ii) the implicit ξ dependence of $\mathcal{E}_A$ in $\hat{A}$ (iii) the explicit ξ dependence of $\mathcal{O}^{-1}$ in the B-field kinetic energy term and (iv) the implicit ξ dependence of auxiliary fields B, ψ, $\overline{\psi}$ and (v) finally from the measure $\mu(\xi)$. Of these, contribution (iv) vanishes since the auxiliary fields satisfy $\frac{\delta S}{\delta \psi}|_{\psi=\psi[\phi]}$=0.

The first contribution reads

$$(\mathrm{I})=\langle\langle \frac{i}{2\xi} \int d^4x(\partial.\hat{A}^a)^2\rangle\rangle=\langle\langle \frac{i}{2\xi}\int d^4x\, \partial.A^a\, \mathcal{E}_R^{-2}\, \partial.A^a\rangle\rangle \qquad (3.6)$$

[with $\mathcal{E}_R^{-2} = e^{\frac{\partial^2}{\Lambda\xi}}$]

The implicit ξ dependence of $\hat{A}$ in ΔS contributes

(IIA)=$-\frac{i(1-\xi)}{2} << \int d^4x \frac{\partial}{\partial\xi}(\partial.\hat{A}^a)^2 >>$ (3.7)
$=-\frac{i(1-\xi)}{2\Lambda^2\xi^2} << \int d^4x\, (\partial.\hat{A}^a)\, e^{-\frac{\partial^2}{2\Lambda^2\xi}}\, \partial^2(\partial.A^a) >>$ (3.8)

and it vanishes at ξ=1.

The contribution from the implicit dependence on ξ of $\hat{A}$ and $\mathcal{O}^{-1}$ in the B-field kinetic terms can be computed straightforwardly. The result reads

(III)=$-\frac{i}{2\Lambda^2\xi} << \int d^4x\, (\partial.\hat{A}^a)\, e^{-\frac{\partial^2}{2\Lambda^2\xi}}\, \partial^2(\partial.A^a) >>+$
$\frac{i}{2\xi} << \int d^4x\, (\partial.B^a)\frac{1}{1-e^{-\frac{\partial^2}{\Lambda^2\xi}}}(\partial.B^a) >>-$
$\frac{i}{2\Lambda^2\xi^2} << \int d^4x\, (\partial.B^a)\, e^{-\frac{\partial^2}{\Lambda^2\xi}}\, \partial^2(\frac{1}{1-e^{-\frac{\partial^2}{\Lambda^2\xi}}})^2\, (\partial.B^a) >>$ (3.9)

The $(\partial.A^a)^2$ type terms in I and III combine to give

$\frac{i}{2\xi}\int d^4x\, (\partial.A^a)\, \mathcal{E}_R^{-2}[1-\frac{\partial^2}{\Lambda^2\xi}]\, (\partial.A^a)$ (3.10)

At ξ=1, these can be simplified using the identity (A.5) in Appendix derived using the BRS WT identity. We note that as far as Green's functions with external gauge fields are concerned, we can set terms $\sim\langle\langle P(\xi)\overline{\eta}^a \frac{\delta S}{\delta\overline{\eta}^a}\rangle\rangle$ to zero as shown in III of the appendix.We are then left with

$-\frac{i}{2}\int d^4x\, \mathcal{E}_R^{-2}[1-\frac{\partial^2}{\Lambda^2\xi}]\, \partial.A^a(x)\overline{\eta}^a(x)\ \int d^4y\, J^{b\mu}(\partial_\mu\eta^b - gf^{bcd}\mathcal{E}^2(A_\mu^c + B_\mu^c)(\eta^d + \psi^d))$ (3.11)

Next we simplify the $(\partial.B^a)^2$ type terms using the relation
$(\partial.B^a)=\frac{e^{\frac{\partial^2}{\Lambda^2\xi}}}{\Lambda^2}[\partial.\frac{\delta\hat{S}_\xi}{\delta A^a} - \frac{e^{\frac{\partial^2}{\Lambda^2\xi}}}{\xi}\partial^2(\partial.A^a)]$ (3.12)

We note that these terms together simplify to yield

$\xi\frac{\partial W}{\partial\xi} = \frac{i}{2\xi}\int d^4x << \partial.\frac{\delta\hat{S}_\xi}{\delta A^a}\hat{P}_1(\xi)\partial.\frac{\delta\hat{S}_\xi}{\delta A^a} >>$
$-\frac{i}{\xi}\int d^4x << \partial.\frac{\delta\hat{S}_\xi}{\delta A^a}\hat{P}_2(\xi)\partial.A^a >>$
$+\frac{i}{2\xi}\int d^4x << \partial.A^a\hat{P}_3(\xi)\partial.A^a >>$ (3.13)

where

$$\hat{P}_1(\xi) = \hat{P}(\xi)\frac{e^{\frac{2\partial^2}{\Lambda^2\xi}}}{\Lambda^4}$$

$$\hat{P}_2(\xi) = \hat{P}(\xi)\frac{e^{\frac{3\partial^2}{\Lambda^2\xi}}}{\Lambda^4}$$

$$\hat{P}_3(\xi) = \hat{P}(\xi)\frac{e^{\frac{4\partial^2}{\Lambda^2\xi}}}{\xi^2}\frac{\partial^2}{\Lambda^2}\frac{\partial^2}{\Lambda^2}$$

$$\hat{P}(\xi)=\frac{1}{1-e^{\frac{-\partial^2}{\Lambda^2\xi}}}\ [1-\frac{\partial^2}{1-e^{\frac{-\partial^2}{\Lambda^2\xi}}}\frac{e^{\frac{-\partial^2}{\Lambda^2\xi}}}{\Lambda^2\xi}]$$

The terms in (3.13) involving $\hat{P}_3(\xi)$ can be simplified as those in (3.10) above and adds up to a term of the same form as (3.11). The second term on the right hand side in (3.13) can be simplified using (A.8). The residual term in (A.8) can be simplified using (A.5) as done earlier. (3.13) can be further simplified at ξ=1 as done in (A.10)-(A.12).

Combining all contributions together we find

$$\xi\frac{\partial W}{\partial\xi}|_{\xi=1,\chi=\chi=0} =<< -\frac{i}{2}\int d^4x\{\mathcal{E}_R^{-2}(1-\frac{\partial^2}{\Lambda^2})+\hat{P}_3(1)+2ig^2N\hat{P}_2(1)\mathcal{M}\}\ \partial.A^a(x)\overline{\eta}^a(x) \int d^4y\ J^{b\mu}(y)\ \{\partial_\mu\eta^b - gf^{bcd}\mathcal{E}_A^2(A_\mu^c+B_\mu^c)(\eta^d+\psi^d)\}\ >>+\langle\langle F\rangle\rangle+\text{measure contribution.} \qquad (3.14)$$

where the Jacobian term F reads

$$F=\frac{3g^2N}{4}[\int\frac{d^4k}{(2\pi)^4}\mathrm{f}(k^2)k^2]\int\frac{d^4p}{(2\pi)^4}\ A_a(\mathrm{p}).A_a(\text{-p}) \qquad (3.15)$$

where

$$f(k^2)=[1+\frac{k^2e^{\frac{k^2}{\Lambda^2}}}{\Lambda^2(1-e^{\frac{k^2}{\Lambda^2}})}]\ \frac{1}{1-e^{\frac{k^2}{\Lambda^2}}}\frac{e^{-\frac{2k^2}{\Lambda^2}}}{\Lambda^4} \qquad (3.16)$$

For arriving at the measure contribution, unlike other contributions, we need the form of the nonlocal BRS transformations for an arbitrary ξ. These are reproduced in Appendix B. We note that the nonlocal BRS and the trivial transformations do not anymore add up to a form where the following two desirable properties convenient for formulating WT identities hold:
(i) δA_μ involves the same combination that is involved in the ghost Lagrangian (ii) $\delta\overline{\eta}$ involves only ∂.A. If we define $\mu(\xi)$ with respect to either

(A) nonlocal BRS of (B.1) or (B) resultant nonlocal BRS of (B.3), we have verified that the measure contribution to (3.14) cannot cancel the Jacobian contribution of (3.15). This cancellation has to be valid in the regularized theory (i.e. for any finite Λ) and we, in particular, draw attention to the fact that $\langle\langle\xi\frac{\partial\mu}{\partial\xi}\rangle\rangle$ contains operators that have arbitrary order derivatives of A while F does not.[Note the form of μ in (2.24)for ξ=1].
Finally, we wish to elaborate upon a shortcoming of the Feynman gauge treatment itself. As elaborated in 2C, the Lagrangian one starts with (of (2.19)) is actually for the case when the unrenormalized paprameter ξ_0=1. When one loop renormalization is carried out, the Lagrangian, when expressed in terms of renormalized fields, now does not retain its form of (2.19). So when we try to extend the treatment to two loops, we have the necessity for the treatment for arbitrary ξ even in this case. This, as presented here, cannot be done along the lines of this section.

IV An Alternate way of regularization that preserves ξ independence

In this section, we shall present a way of regularization that is at once simpler and leads to WT identities that would imply the ξ independence of S-matrix elements (where they exist).We shall construct the relevent non-local BRS transformation that leads to the simpler form of the WT identity. A similar regularization has already been applied to QED[7].

We recall that the local action of the nonabelian gauge theory with an arbitrary ξ. It is expressed as

$$S_\xi = S_F + \Delta S \qquad (4.1)$$

where S_F is the Feynman gauge local action.

We introduce the smeared field operators that depend only on the quadratic form in S_F. Hence, we have for an arbitrary ξ

$$\hat{A'}_\mu = (\mathcal{E}_F^{-1})_{\mu\nu}A^\nu = e^{\frac{\partial^2}{2\Lambda^2}}A_\mu \qquad (4.2a)$$

$\hat{\eta}' = \mathcal{E}_F^{-1}\eta = e^{\frac{\partial^2}{2\Lambda^2}}\eta$ (4.2b)

We note $\mathcal{E}_F = \mathcal{E}_\eta$ here.

We write down the non-local action following the same rules as those in [4] otherwise.

Explicitly,

$S'_\xi = S'_F + \Delta S'$ (4.3)

with
S'_F[A',η',$\overline{\eta'}$]=$\int d^4x\{$-$\frac{1}{2}\partial_\mu A'^a_\nu\, \partial^\mu A'^{a\nu}$-$\frac{1}{2}B'^a_\mu \mathcal{O}^{-1} B'^{a\mu}$-$\partial_\mu \overline{\eta'}^a\, \partial^\mu \eta'^a$+$\overline{\psi'}^a \mathcal{O}^{-1}\psi'^a\}$
+I[A'+B',$\eta' + \psi'$,$\overline{\eta'} + \overline{\psi'}$] (4.4)
and

ΔS=$\frac{1}{2}(1-\frac{1}{\xi})\int d^4x(\partial.A'^a)^2$ (4.5)

We note that in (4.4) the kinetic term for the auxiliary field B involves $\mathcal{O} = \frac{\mathcal{E}_F^2 - 1}{\mathcal{F}_F}$ that is ξ independent. We further note that the **form** of the relations between auxiliary fields (B',ψ',$\overline{\psi}'$) and A',η',$\overline{\eta}'$ is same as in Feynman gauge as $\Delta\hat{S}$ does not contribute to these relations.[4]

Now, consider the change in the effective action S_ξ under a field transformation

$\delta\hat{S}_\xi = \delta\mathcal{S}[\phi,\psi[\phi]]$=$\delta\mathcal{S}[\phi,\psi]|_{\psi=\psi[\phi]}$ (4.6)

Thus

[4]We note that the present regularization does not preserve the properties of (2.13),(2.18). We also note that the shadow and the barred propagators in this regularization do not add up to the local one; but differ from it by "gauge terms" (that vanish as $\Lambda \longrightarrow \infty$). This does not constitute a problem however. We can look upon this as a part of existing freedom in defining the local theory itself that exists on acount of gauge invariance.

$\delta\hat{S}_\xi = [\delta\phi\frac{\delta\mathcal{S}}{\delta\phi} + \delta\psi\frac{\delta\mathcal{S}}{\delta\psi}]_{\psi=\psi[\phi]}$ (4.7)
The second term vanishes by the defining relation for ψ.
Thus

$$\delta\hat{S}'_\xi = \delta\phi\frac{\delta\hat{S}'_F}{\delta\phi} + \delta\phi\frac{\delta\Delta\hat{S}'}{\delta\phi} \quad (4.8)$$

Now consider the non-local BRS transformations of the Feynman gauge non-local action.

$$\hat{\delta}A^a_\mu = \mathcal{E}^2_F[\partial_\mu(\eta+\psi)^a - gf^{abc}(A+B)^b_\mu(\eta+\psi)^c]\delta\varsigma \quad (4.9a)$$

$$\hat{\delta}\eta^a = -\frac{g}{2}f^{abc}\mathcal{E}^2_\eta(\eta+\psi)^b(\eta+\psi)^c\delta\varsigma \quad (4.9b)$$

$$\hat{\delta}\overline{\eta}^a = -\mathcal{E}^2_\eta(\partial.A^a + \partial.B^a)\delta\varsigma \quad (4.9c)$$

We know that since $\hat{S}_F[\phi,\psi[\phi]]$ is exactly of the same form as in the Feynman gauge, it is invariant under the Feynman gauge non-local BRS transformations of (4.9). On the other hand we find by explicit calculation

$$\delta(\Delta\hat{S}) = (1-\frac{1}{\xi})\int d^4x[\mathcal{E}^2(\partial.A^a)]\frac{\delta\hat{S}_\xi}{\delta\eta^a}\delta\varsigma \quad (4.10)$$

This change in $\Delta\hat{S}$ can be canceled by an additional change

$$\hat{\delta}'\overline{\eta}^a = (1-\frac{1}{\xi})\mathcal{E}^2(\partial.A^a)\delta\varsigma \quad (4.11)$$

In addition, we also note the dynamically irrelevent symmetries mentioned in the theorem (B.4). It reads

$$\hat{\delta}^o A^a_\mu = [(1-\mathcal{E}^2)\partial_\mu\eta^a - \mathcal{E}^2\partial_\mu\psi^a]\delta\varsigma \quad (4.12a)$$

$$\hat{\delta}^o\eta^a = 0 \quad (4.12b)$$

$$\hat{\delta}^o\overline{\eta}^a = [\mathcal{E}^2(\partial.B^a) - \frac{1}{\xi}(1-\mathcal{E}^2)\partial.A^a]\delta\varsigma \quad (4.12c)$$

This is an invariance of $\hat{S}_\xi$ follows from theorem (B.4) and has been verified by explicit evaluation.

We add the transformations of (4.9),(4.11)and (4.12) to obtain the final non-local BRS symmetry of the non-localized action for an arbitrary ξ. It reads, for an arbitrary ξ:

$$\hat{\delta}A^a_\mu = [\partial_\mu\eta^a - gf^{abc}\mathcal{E}^2(A+B)^b_\mu(\eta+\psi)^c]\delta\varsigma \qquad (4.13a)$$

$$\hat{\delta}\eta^a = -\frac{g}{2}f^{abc}\mathcal{E}^2(\eta+\psi)^b(\eta+\psi)^c\delta\varsigma \qquad (4.13b)$$

$$\hat{\delta}\overline{\eta}^a = -\frac{1}{\xi}\partial.A^a\delta\varsigma \qquad (4.13c)$$

This now leads to the nonlocal BRS WT identity valid for an arbitrary ξ of (A.4). We note here that as the Jacobian for the nonlocal trasformation (4.13) is ξ-independent by construction, μ can be taken to be independent of ξ.

We now obtain the value of $\xi\frac{dW}{d\xi}|_{\chi=\overline{\chi}=0}$, for an arbitrary ξ, in this formulation. We note the ξ dependence now entirely comes from the explicit ξ dependence of $\Delta\hat{S}$; since the regulators $\mathcal{E}$, $\mathcal{O}^{-1}$ are independent of ξ. We, thus, have

$$\xi\frac{\partial W}{\partial\xi} = \frac{i}{2\xi} << \int d^4x\,\partial.A^a\,\mathcal{E}_F^{-2}\,\partial.A^a >>$$

The above can be effectively simplified using (A.5) [which now holds for an arbitrary ξ], (A.6) and (A.9) to lead to

$$\xi\frac{\partial W}{\partial\xi}|_{J=J_{phy},\chi=\overline{\chi}=0} =<< -\frac{i}{2}\int d^4x\,[\mathcal{E}_F^{-2}\,\partial.A^a(x)]\eta^a(x)$$
$$\int d^4y\,J^{b\mu}(y)\{\partial_\mu\eta^b(y) - gf^{bcd}\mathcal{E}^2(A+B)^c_\mu(\eta+\psi)^d\} >> \qquad (4.15)$$

The above WT identity is the key to the ξ-independence of S-matrix elements (or quantities derived from them) wherever they exist. We note that in such cases, the discussion of ξ-independence should run entirely parallel to that in Ref.[15]. One can adopt a limiting procedure of carrying out renormalization at an off-shell point p^2=-μ^2 and then take the limit $\mu^2\longrightarrow 0$ in the final result. We expect that when a similar regularization applied in the SBGT, the resulting WT identity similar to (4.15) will lead to the ξ-

independence of the S-matrix elements that exist.

Appendix A

In this appendix, we shall derive the auxiliary equations needed in simplifying $\xi\frac{\partial W}{\partial \xi}$. We consider the field transformation, possibly nonlocal, [ϵ infinitesimal and ϕ stands collectively for A, η and $\overline{\eta}$]

$$\phi \to \phi + \epsilon F[\phi] \qquad \text{(A.1)}$$

in the field variables in W[J, χ, $\overline{\chi}$] to obtain the generalized equation of motion:

$$\langle\langle \int d^4\mathrm{x}\{\mathrm{i}\textstyle\sum_i F_i[\phi]\ \frac{\delta \hat{S}_\xi}{\delta\phi_i} + \sum_i J_i F_i[\phi] + \mathcal{F} + \sum_i F_i \frac{\delta}{\delta\phi_i}\ln\mu\}\ \rangle\rangle = 0. \qquad \text{(A.2)}$$

Here, $\sum_i J_i F_i[\phi]$ collectively denotes the source terms and $\epsilon\mathcal{F}$ stands for the Jacobian (minus one) for the field transformation (A.1) viz

$$\mathcal{F} = \int d^4\mathrm{x} \frac{\delta F[\phi]}{\delta\phi(y)}|_{x=y} \qquad \text{(A.3)}$$

Note that the measure factor μ in Feynman gauge has been chosen so that the last two terms on the right hand side of (A.2) vanish for the (modified) nonlocal BRS transformations of (4.13). Thus, as a special case, we have the BRS nonlocal WT identity resulting from the surviving second term in (A.2)

$$\langle\langle \int d^4\mathrm{x}[J^{a\mu}\ \hat{\delta}A^a_\mu + \overline{\chi}^a\ \hat{\delta}\eta^a + \hat{\delta}\overline{\eta}^a\ \chi^a]\rangle\rangle = 0 \qquad \text{(A.4)}$$

Let P(ξ) be any arbitrary differential operator that may depend on ξ but not on the fields. We operate by $\mathcal{E}_A^{-2}\mathrm{P}(\xi)\partial^\alpha \frac{\delta}{\delta J^{p\alpha}(y)} \frac{\delta}{\delta\chi^c(y)}$ on (A.4) and put χ=0=$\overline{\chi}$ to obtain,

$$\begin{aligned}&-\mathrm{i}\tfrac{1}{\xi}\langle\langle[\mathrm{P}(\xi)\mathcal{E}_A^{-2}\partial.A^p\partial.A^c] - \mathrm{P}(\xi)\overline{\eta}^c \tfrac{\delta S}{\delta\overline{\eta}^p}\rangle\rangle = \\ &\langle\langle \int d^4z J^{\mu b}(z)[\partial_\mu\eta^b - gf^{bcd}{\mathcal{E}_A}^2[(A+B)_\mu{}^c(\eta+\xi)^d]](z) \\ &{\mathcal{E}_A}^{-2}P(\xi)\partial.A^p(x)\overline{\eta}^c(x)\rangle\rangle \qquad \text{(A.5)}\end{aligned}$$

Further we note that the term of the form $\langle\langle \int d^4x\ \partial^\mu J_\mu(x)\mathrm{G}[\phi]\rangle\rangle$ do not contribute to the Green's function with external gauge boson lines with the

physical polarization vectors since ϵ.k=0.

We express this by saying $\langle\langle\int d^4x\ \partial^\mu J_\mu(x)\mathrm{G}[\phi]\rangle\rangle|_{phy}$=0. (A.6)
[In using the subscript 'phy' we do not necessarily imply mass shell limit however.]
We shall need a set of results derivable from (A.2)

(I) We let F[ϕ] be linear in the fields. Then $\mathcal{F}$ is field independent and can be dropped while evaluating $\xi\frac{\partial}{\partial\xi}$ of n-point Green's functions.

(II) With $F^a_\mu[\phi]$=$\partial_\mu\hat{P}_2(\xi)\,\partial.A^a$ (for δA_μ) and F_η=$F_{\overline{\eta}}$=0, we then obtain

$$\langle\langle\int d^4x\{\mathrm{i}\partial.A^a(\mathrm{x})\hat{P}_2(\xi)\,\partial.\frac{\delta\hat{S}_\xi}{\delta A^a}+F^a_\mu[A]\frac{\delta}{\delta A^a_\mu}\ln\ \mu+\mathrm{i}J^a_\mu(x)\partial^\mu\hat{P}_2(\xi)\,\partial.A^a(x)\}\rangle\rangle=0 \quad (A.7)$$

The measure factor at $\xi = 1$ has been evaluated in $[K_1]$ and is given by [2.24]. Using it, we obtain, at $\xi = 1$ and with J=J_{phy}

$$\langle\langle\int d^4x[i\partial.A^a\hat{P}_2(\xi)\partial.\frac{\delta\hat{S}_\xi}{\delta A^a} - g^2 f^{acd}f^{bcd}F^a{}_\mu[A]\mathcal{M}A^{b\mu}(x)]\rangle\rangle|_{phy}=0 \quad (A.8)$$

(III) We let $F_A = 0 = F_\eta$ and $F_{\overline{\eta}}{}^a = P(\xi)\overline{\eta}^a$ which is a linear transformation. Noting that μ does not depend on $\overline{\eta}$ and (I) above, we obtain that at $\xi = 1$ and $\chi = \overline{\chi} = 0$,

$$\langle\langle\int d^4x\mathrm{P}(\xi)\ \overline{\eta}^a\frac{\delta\hat{S}_\xi}{\delta\overline{\eta}^a(x)}\rangle\rangle=0 \quad (A.9)$$

(IV) Finally, we let $F^a_\mu[\phi]$=$\partial_\mu\hat{P}_1(\xi)\ \partial.\frac{\delta\hat{S}_\xi}{\delta A^a(x)}$. For physical sources, we find

$$\mathrm{i}\langle\langle\int d^4x\partial.\frac{\delta\hat{S}_\xi}{\delta A^a}\hat{P}_1(\xi)\partial.\frac{\delta\hat{S}_\xi}{\delta A^a}\rangle\rangle|_{J=J_{phy}}=-\langle\langle\mathcal{F}+\frac{\delta}{\delta A^a_\mu}ln\mu F^a_\mu\rangle\rangle \quad (A.10)$$

For $\xi = 1$, we find that the term coming from the measure equals

$$\int d^4xg^2N\langle\langle\partial.\frac{\delta\hat{S}_\xi}{\delta A^a}\hat{P}_1(\xi)\mathcal{M}\partial.A^a\rangle\rangle+\mathrm{O}(g^3) \quad (A.11)$$

This can be reduced further by using (A.8) to obtain

$$\langle\langle\frac{\delta ln\mu}{\delta A^a_\mu}F^a_\mu\rangle\rangle|_{\xi=1,J=J_{phy}}=\mathrm{O}\ (g^4) \quad (A.12)$$

Appendix B

In this appendix, we shall write down the nonlocal BRS transformations for the case of an arbitrary ξ following the general formalism of [4]. We have, for the gauge fields,

$\mathcal{F}_{\mu\nu}= \eta_{\mu\nu}\partial^2$-(1-$\frac{1}{\xi}$)$\partial_\mu\partial_\nu$

for the Euclidean formulation $\eta_{\mu\nu}$=diag(-1,-1,-1,-1).

We define

$$(\mathcal{E}_A)_{\mu\nu}=(e^{\frac{\mathcal{F}}{2\Lambda^2}})_{\mu\nu} \tag{B.1}$$

We note $\partial^\mu(\mathcal{E}_A)_{\mu\nu}=e^{-\frac{\partial^2}{2\Lambda^2\xi}}\partial_\nu\equiv{\mathcal{E}_A}^0\partial_\nu$.

For the ghost case, we continue to define $\mathcal{E}_\eta=e^{-\frac{\partial^2}{2\Lambda^2}}\neq{\mathcal{E}_A}^0$

Then the nonlocal BRS transformations read:

$$\hat{\delta}A^a_\mu = ({\mathcal{E}_\mathcal{A}}^2)_\mu{}^\nu[\partial_\nu(\eta^a+\psi^a)-gf^{abc}(A^b_\nu+B^b_\nu)(\eta^c+\psi^c)]\delta\varsigma$$

$$\hat{\delta}\eta^a = \text{-}\tfrac{g}{2}f^{abc}{\mathcal{E}_\eta}^2(\eta^b+\psi^b)(\eta^c+\psi^c)\delta\varsigma \tag{B.2}$$

$$\hat{\delta}\overline{\eta}^a = -{\mathcal{E}_\eta}^2\partial_\mu(A^{a\mu}+B^{a\mu})\tfrac{\delta\varsigma}{\xi}$$

The trivial transformations, on the other hand, read

$$\hat{\delta}^o{A_\mu}^a=\rho[(1-{\mathcal{E}_\eta}^2)\partial_\mu\eta^a\text{-}{\mathcal{E}_\eta}^2\partial_\mu\psi^a]\delta\varsigma$$

$$\hat{\delta}^o\eta^a = 0 \tag{B.3}$$

$$\hat{\delta}^o\overline{\eta}^a=\rho[{\mathcal{E}_\mathcal{A}}^{0\,2}(\partial.B^a)-(1-{\mathcal{E}_\mathcal{A}}^{0\,2})\partial.A^a]\delta\varsigma$$

where ρ is any constant. We note that in the first of (B.2) '$\mathcal{E}_A$' appears in $\hat{\delta}{A_\mu}^a$, while is the first of (B.3) '$\mathcal{E}_\eta$ ' appears in $\hat{\delta}^0{A_\mu}^a$. In the case of ξ=1, $\mathcal{E}_\eta$=$\mathcal{E}_A$=${\mathcal{E}_A}^0$. Then with $\rho=1$, the first of (B.2) and (B.3) added together lead to $\hat{\delta}$'A which contains the same combination of terms present in the ghost Lagrangian and this leads to the simplification in the expression for $\xi\frac{\partial W}{\partial\xi}$. This no longer happens for $\xi\neq1$. Similarly, the last of (B.2) and (B.3) now contain different regulators '${\mathcal{E}_\eta}^2$ ' and '${\mathcal{E}_\mathcal{A}}^{0\,2}$' respectively. So, even with $\rho=\frac{1}{\xi}$, they do not lead to the cancellation of ∂.B terms. Moreover, note that

the value of ρ needed in the first of (B.3) needed for a '**near**' cancellation of unwanted terms does not agree with the value of ρ in the last of (B.3) for a 'near' cancellation of ∂.B terms. As a result of this, for $\xi \neq 1$, we do not have the simplified treatment of [4] available in the standard treatment. This holds, even if we try to modify (B.2).
We could define measure $\mu(\xi)$ with respect to (B.2)+(B.3) with either $\rho = 1$ or $\rho = \frac{1}{\xi}$ and, in either case, we find that $\mu(\xi)$ must contain terms that cannot cancel the term F of (3.15).

References:

1: G. 't Hooft and M. Veltman Nucl. Phys. B33, 189 (1972)
2: E. D. Evans et al, Phys Rev D43, 499 (1991)
3: See e.g, D. Z. Freedman et al, Nucl. Phys. B395, 454(1993)
4: G. Kleppe and R. P. Woodard, Nucl. Phys. B388, 81 (1992)
5: G. Kleppe and R. P. Woodard, Ann. Phys. (N.Y.) 221, 106 (1993)
6: See e.g. [7] and references therein
7: M. A. Clayton Gauge Invariance in nonlocal regularized QED. (Toronto U.) UTPT-93-14, Jul 1993. [hep-th/9307089]
8: S. D. Joglekar and G. Saini, Z. Phys. C.76, 343-353 (1997)
9: S. D. Joglekar The condition 0 $\langle$ Z $\langle$ and an intrinsic mass scale in quantum field theory. Oct1999. 9pp. and hep-th/0003077
10: S. D. Joglekar Understanding of the renormalization programme in a mathematically rigorous framework and an intrinsic mass scale. Dec 1999. 18pp. and hep-th/0003104
11: J. Paris and W. Troost: hep-th/9607215
12: See e.g. E. Abers and B. W. Lee Phys. Rep. 9C(1973),1
13: S. D. Joglekar and G. Saini (Unpublished)
14: A. Basu and S. D. Joglekar [in preparation]
15: See e.g. B. W. Lee in "Methods in Field Theory" Les Houches 1975 Editor: R. Balian and J. Zinn-Justin.